

# Package ‘backShift’

October 12, 2022

**Type** Package

**Title** Learning Causal Cyclic Graphs from Unknown Shift Interventions

**Version** 0.1.4.3

**Date** 2020-05-06

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**Depends** R (>= 3.1.0)

**Imports** methods, clue, igraph, matrixcalc, reshape2, ggplot2, MASS

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**Description** Code for 'backShift', an algorithm to estimate the connectivity matrix of a directed (possibly cyclic) graph with hidden variables. The underlying system is required to be linear and we assume that observations under different shift interventions are available. For more details, see <[arXiv:1506.02494](https://arxiv.org/abs/1506.02494)>.

**License** GPL

**LazyData** true

**Suggests** knitr, pander, fields, testthat, pcalg, rmarkdown

**VignetteBuilder** knitr

**URL** <https://github.com/christinaheinze/backShift>

**BugReports** <https://github.com/christinaheinze/backShift/issues>

**RoxygenNote** 7.0.2

**NeedsCompilation** yes

**Repository** CRAN

**Date/Publication** 2020-05-06 11:30:03 UTC

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backShift	<i>Estimate connectivity matrix of a directed graph with linear effects and hidden variables.</i>
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### Description

This function estimates the connectivity matrix of a directed (possibly cyclic) graph with hidden variables. The underlying system is required to be linear and we assume that observations under different shift interventions are available. More precisely, the function takes as an input an (nxp) data matrix, where  $n$  is the sample size and  $p$  the number of variables. In each environment  $j$  ( $j$  in  $\{1, \dots, J\}$ ) we have observed  $n_j$  samples generated from

$$X_j = X_j * A + c_j + e_j$$

(in case of cycles this should be understood as an equilibrium distribution). The  $c_j$  is a  $p$ -dimensional random vector that is assumed to have a diagonal covariance matrix. The noise vector  $e_j$  is assumed to have the same distribution in all environments  $j$  but is allowed to have an arbitrary covariance matrix. The different intervention settings are provided to the method with the help of the vector ExpInd of length  $n = (n_1 + \dots + n_j + \dots + n_J)$ . The goal is to estimate the connectivity matrix  $A$ .

### Usage

```
backShift(X, ExpInd, covariance=TRUE, ev=0, threshold =0.75, nsim=100,
          sampleSettings=1/sqrt(2), sampleObservations=1/sqrt(2),
          nodewise=TRUE, tolerance = 10^(-4), baseSettingEnv = 1,
          verbose = FALSE)
```

### Arguments

X	A (nxp)-dimensional matrix (or data frame) with n observations of p variables.
ExpInd	Indicator of the experiment or the intervention type an observation belongs to. A numeric vector of length n. Has to contain at least three different unique values.
covariance	A boolean variable. If TRUE, use only shift in covariance matrix; otherwise use shift in Gram matrix. Set only to FALSE if at most one variable has a non-zero shift in mean in the same setting (default is TRUE).
ev	The expected number of false selections for stability selection. No stability selection computed if ev=0. Defaults to ev=0.

threshold	The selection threshold for stability selection (has to be between 0.5 and 1). Edges which are selected with empirical proportion higher than threshold will be retained.
nsim	Number of resamples taken (if using stability selection).
sampleSettings	The proportion of unique settings to resample for each resample; has to be in [0,1].
sampleObservations	The fraction of all samples to retain when subsampling (no replacement); has to be in [0,1].
nodewise	If FALSE, stability selection retains for each subsample the largest overall entries in the connectivity matrix. If TRUE, values are ordered row- and node-wise first and then the largest entries in each row and column are retained. Error control is valid (under exchangeability assumption) in both cases. The latter setting TRUE is perhaps more robust and is the default.
tolerance	Precision parameter for <code>ffdiag</code> : the algorithm stops when the criterium difference between two iterations is less than tolerance. Default is $10^{-4}$ .
baseSettingEnv	Index for baseline environment against which the intervention variances are measured. Defaults to 1.
verbose	If FALSE, most messages are suppressed.

## Value

A list with elements

Ahat	The connectivity matrix where entry (i,j) is the effect pointing from variable i to variable j.
AhatAdjacency	If $ev > 0$ , the connectivity matrix retained by stability selection. Entries give the rounded percentage of times the edge has been retained (and 0 if below the critical threshold).
varianceEnv	The estimated interventions variances up to an offset. <code>varianceEnv</code> is a (Gxp)-dimensional matrix where G is the number of unique environments. The ij-th entry contains the difference between the estimated intervention variance of variable j in environment i and the estimated intervention variance of variable j in the base setting (given by input parameter <code>baseSettingEnv</code> ).

## Author(s)

Christina Heinze-Deml <heinzdeml@stat.math.ethz.ch>

## References

Dominik Rothenhausler, Christina Heinze, Jonas Peters, Nicolai Meinshausen: backShift: Learning causal cyclic graphs from unknown shift interventions. Advances in Neural Information Processing Systems (NIPS) 28, 2015. arXiv: <http://arxiv.org/abs/1506.02494>

## See Also

[ICP](#) and [hiddenICP](#) for reconstructing the parents of a variable under interventions on all other variables. [getParents](#) and [getParentsStable](#) from the package [CompareCausalNetworks](#) to estimate the connectivity matrix of a directed causal graph, using various possible methods (including `backShift`).

## Examples

```
## Simulate data with connectivity matrix A

seed <- 1
# sample size n
n <- 10000
# 3 predictor variables
p <- 3
A <- diag(p)*0
A[1,2] <- 0.8
A[2,3] <- -0.8
A[3,1] <- 0.8

# divide data into 10 different environments
G <- 10

# simulate
simulation.res <- simulateInterventions(
  n, p, A, G, intervMultiplier = 2,
  noiseMult = 1, nonGauss = FALSE,
  fracVarInt = 0.5, hidden = TRUE,
  knownInterventions = FALSE,
  simulateObs = TRUE, seed)

environment <- simulation.res$environment
X <- simulation.res$X

## Compute feedback estimator with stability selection

network <- backShift(X, environment, ev = 1)

## Print point estimates and stable edges

# true connectivity matrix
print(A)
# point estimate
print(network$Ahat)
# shows empirical selection probability for stable edges
print(network$AhatAdjacency)
```

---

bootstrapBackShift	<i>Computes a simple model-based bootstrap confidence interval for success of joint diagonalization procedure. The model-based bootstrap approach assumes normally distributed error terms; the parameters of the noise distribution are estimated with maximum likelihood.</i>
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### Description

Computes a simple model-based bootstrap confidence interval for success of joint diagonalization procedure. The model-based bootstrap approach assumes normally distributed error terms; the parameters of the noise distribution are estimated with maximum likelihood.

### Usage

```
bootstrapBackShift(
  Ahat,
  X,
  ExpInd,
  nrep,
  alpha = 0.05,
  covariance = TRUE,
  baseInd = 1,
  tolerance = 0.001,
  verbose = FALSE
)
```

### Arguments

Ahat	Estimated connectivity matrix returned by backShift.
X	A (n <sub>xp</sub> )-dimensional matrix (or data frame) with n observations of p variables.
ExpInd	Indicator of the experiment or the intervention type an observation belongs to. A numeric vector of length n. Has to contain at least three different unique values.
nrep	Number of bootstrap samples.
alpha	Significance level for confidence interval.
covariance	A boolean variable. If TRUE, use only shift in covariance matrix; otherwise use shift in Gram matrix. Set only to FALSE if at most one variable has a non-zero shift in mean in the same setting (default is TRUE).
baseInd	Index for baseline environment against which the intervention variances are measured. Defaults to 1.
tolerance	Precision parameter for ffdiag: the algorithm stops when the criterium difference between two iterations is less than tolerance. Default is 10 <sup>(-4)</sup> .
verbose	If FALSE, messages are suppressed.

**Value**

A list with the following elements:

- `bootsSumOffDiags` Vector of length `nrep` with sum of off-diagonal elements after joint diagonalization procedure in each of the bootstrap samples.
- `sumOffDiagsBackShift` Sum of off-diagonal elements after joint diagonalization procedure in original estimation.
- `jointDiagSuccess` TRUE if `sumOffDiagsBackShift` lies within bootstrap confidence interval.
- `lower` Lower bound of bootstrap confidence interval.
- `upper` Upper bound of bootstrap confidence interval.
- `lowerBasic`  $\alpha/2$  quantile of empirical bootstrap distribution.
- `upperBasic`  $1 - \alpha/2$  quantile of empirical bootstrap distribution.

---

`computeDiagonalization`

*Computes the matrix  $\Delta\Sigma_{c,j}$  resulting from the joint diagonalization for a given environment (cf. Eq.(7) in the paper). If the joint diagonalization was successful the matrix should be diagonal for all environments  $j$ .*

---

**Description**

Computes the matrix  $\Delta\Sigma_{c,j}$  resulting from the joint diagonalization for a given environment (cf. Eq.(7) in the paper). If the joint diagonalization was successful the matrix should be diagonal for all environments  $j$ .

**Usage**

```
computeDiagonalization(estConnectivity, X, env, whichEnv, main = NULL)
```

**Arguments**

<code>estConnectivity</code>	Estimate for connectivity matrix returned by <code>backShift</code> .
<code>X</code>	Data matrix
<code>env</code>	Indicator of the experiment or the intervention type an observation belongs to (a numeric vector of length <code>n</code> ).
<code>whichEnv</code>	Indicator for the environment for which the matrix $\Delta\Sigma_{c,j}$ should be computed.
<code>main</code>	Optional title for plot; defaults to <code>paste("Env.", whichEnv)</code>

---

`exampleAdjacencyMatrix`*Example adjacency matrix*

---

**Description**

An example for an adjacency matrix  $A$  to be used as input to `simulateInterventions`. The entry  $A_{ij}$  contains the edge from node  $i$  to node  $j$ .

**Usage**

```
data("exampleAdjacencyMatrix")
```

**Format**

A matrix with 10 rows and 10 columns.

**References**

Used in simulations in:

Dominik Rothenhaeusler, Christina Heinze, Jonas Peters, Nicolai Meinshausen (2015): backShift: Learning causal cyclic graphs from unknown shift interventions arXiv preprint: <http://arxiv.org/abs/1506.02494>

**Examples**

```
data("exampleAdjacencyMatrix")
plotGraphEdgeAttr(estimate = exampleAdjacencyMatrix, plotStabSelec = FALSE,
                  labels = colnames(exampleAdjacencyMatrix),
                  thres.point = 0, thres.stab = NULL, main = "True graph")
```

---

`generateA`*Generates a connectivity matrix A.*

---

**Description**

Generates a connectivity matrix  $A$  with cycle product smaller than 1.

**Usage**

```
generateA(p, expNumNeigh, minCoef, maxCoef, cyclic, verbose = FALSE)
```

**Arguments**

p	Number of variables.
expNumNeigh	Expected number of neighbors, to be passed to <code>randDAG</code> .
minCoef	Minimal edge coefficient. The absolute magnitude of the coefficients will be sampled uniformly at random from the range $[minCoef, maxCoef]$ .
maxCoef	Maximal edge coefficient. The absolute magnitude of the coefficients will be sampled uniformly at random from the range $[minCoef, maxCoef]$ .
cyclic	If TRUE, connectivity matrix will contain at least one cycle.
verbose	If TRUE, comments will be printed.

**Details**

If `expNumNeigh` and `maxCoef` are large, function may fail to find a connectivity matrix with cycle product smaller one. In this case, try to lower these parameters.

**Value**

A list with two elements

- A Connectivity matrix
- `sizeCycle` Size of the cycle, if `cyclic` was set to TRUE.

---

metricsThreshold	<i>Performance metrics for estimate of connectivity matrix A.</i>
------------------	---

---

**Description**

Computes various performance metrics for estimate of connectivity matrix A.

**Usage**

```
metricsThreshold(trueA, est, thres = seq(0.01, 1, by = 0.01))
```

**Arguments**

trueA	True connectivity matrix
est	Estimated connectivity matrix
thres	Value at which the point estimate should be thresholded, i.e. edges with coefficients smaller than <code>thres</code> are discarded. Can be a sequence of values.



**Value**

A data frame with the following columns:

- Threshold Value at which point estimate est was thresholded.
- SHD Structural Hamming distance between trueA and est.
- TPR.Recall True positive rate / recall value
- FPR False positive rate
- Precision Precision value

**Examples**

```
# true A
p <- 3
A <- diag(p)*0
A[1,2] <- 0.8
A[2,3] <- -0.8
A[3,1] <- 0.8

# say an estimated connectivity matrix is given by:
A.est <- matrix(rnorm(p*p, 1e-3, 1e-3), ncol = p)
diag(A.est) <- 0
A.est[1,2] <- 0.76
A.est[2,3] <- -0.68
A.est[3,1] <- 0.83

# compute metrics with threshold 0.25
metricsThreshold(A, A.est, thres = 0.25)
```

---

plotDiagonalization     *Plots the joint diagonalization. I.e. if it was successful the matrices should all be diagonal.*

---

**Description**

Plots the joint diagonalization. I.e. if it was successful the matrices should all be diagonal.

**Usage**

```
plotDiagonalization(estConnectivity, X, env, whichEnv, main = NULL)
```

**Arguments**

estConnectivity	Estimate for connectivity matrix returned by backShift.
X	Data matrix
env	Indicator of the experiment or the intervention type an observation belongs to (a numeric vector of length n).

whichEnv	Indicator for the environment to be plotted.
main	Optional title for plot; defaults to paste("Env.", whichEnv)

---

plotGraphEdgeAttr      *Plotting function to visualize directed graphs*

---

## Description

Given a point estimate of the connectivity matrix or the adjacency matrix, this function visualizes the directed graph using `plot.igraph` from the package `igraph`. If a point estimate is plotted, the edges' intensity reflects the magnitude of the coefficients. If the result is an adjacency matrix estimated by stability selection then the edges' width reflects how often an edge was selected and the intensity reflects the magnitude of the coefficients (if this information is also provided).

## Usage

```
plotGraphEdgeAttr(
  estimate,
  plotStabSelec,
  labels,
  thres.point,
  edgeWeights = NULL,
  thres.stab = 0.75,
  main = "",
  edge.color = "blue",
  ...
)
```

## Arguments

estimate	Estimate of connectivity matrix. This can be a point estimate with entry $A_{ij}$ being the estimated edge weight for the edge from node $i$ to node $j$ . Otherwise, it can be the estimated adjacency matrix by a stability selection procedure as in <code>backShift</code> . In this case, the entry $A_{ij}$ indicates how often the edge from node $i$ to node $j$ was selected.
plotStabSelec	Set to TRUE if estimate results from the stability selection procedure. Otherwise, estimate is assumed to be a point estimate.
labels	Variable labels to be displayed in plot.
thres.point	Value at which the point estimate should be thresholded, i.e. edges with coefficients smaller than <code>thres.point</code> are not displayed.
edgeWeights	If stability selection result should be visualized, provide edgeWeights as a (pxp)-matrix to display the magnitude of the coefficients as the intensity of the edges.
thres.stab	Indicate the threshold value that was used in the stability selection procedure. Used to determine the width of the plotted edges.
main	Provide the title of the plot.

edge.color      Color of the edges. Defaults to blue.

...              Optional arguments passed to the plotting function. Consists of igraph-type options like vertex.label.cex, vertex.label.color, edge.arrow.size or vertex.size etc.

@examples # create a matrix A to be visualized p <- 3 A <- diag(p)\*0 A[1,2] <- 0.8 A[2,3] <- -0.8 A[3,1] <- 0.8

# add column names to use as labels for nodes colnames(A) <- c("1", "2", "3")

# plot plotGraphEdgeAttr(estimate = A, plotStabSelec = FALSE, labels = colnames(A), thres.point = 0, thres.stab = NULL, main = "True graph")

### Details

Currently not all options of [igraph](#) are used; additional arguments are ignored.

---

plotInterventionVars    *Plots the estimated intervention variances.*

---

### Description

Plots the estimated intervention variances.

### Usage

```
plotInterventionVars(estIntVars, trueIntVars = NULL, scales_facet = "free")
```

### Arguments

estIntVars      A (Gxp)-dimensional matrix with the estimated intervention variances returned by backShift (as varianceEnv). G is the number of unique environments, p is the number of variables.

trueIntVars     A (Gxp)-dimensional matrix with the true intervention variances if these are known (for simulations). By default this parameter is set to NULL.

scales\_facet    scales argument passed to ggplot's facet\_wrap

---

simulateInterventions    *Simulate data of a causal cyclic model under shift interventions.*

---

### Description

Simulate data of a causal cyclic model under shift interventions.

**Usage**

```
simulateInterventions(
  n,
  p,
  A,
  G,
  intervMultiplier,
  noiseMult,
  nonGauss,
  hiddenVars,
  knownInterventions,
  fracVarInt,
  simulateObs,
  seed = 1
)
```

**Arguments**

n	Number of observations.
p	Number of variables.
A	Connectivity matrix A. The entry $A_{ij}$ contains the edge from node i to node j.
G	Number of environments, has to be larger than two for backShift.
intervMultiplier	Regulates the strength of the interventions.
noiseMult	Regulates the noise variance.
nonGauss	Set to TRUE to generate non-Gaussian noise.
hiddenVars	Set to TRUE to include hidden variables.
knownInterventions	Set to TRUE if location of interventions should be known.
fracVarInt	If knownInterventions is TRUE, fraction of variables that are intervened on in each environment.
simulateObs	If TRUE, also generate observational data.
seed	Random seed.

**Value**

A list with the following elements:

- X (nxp)-dimensional data matrix
- environment Indicator of the experiment or the intervention type an observation belongs to. A numeric vector of length n.
- interventionVar (Gxp)-dimensional matrix with intervention variances.
- interventions Location of interventions if knownInterventions was set to TRUE.
- configs A list with the following elements:

- trueA True connectivity matrix used to generate the data.
- G Number of environments.
- indexObservationalData Index of observational data
- intervMultiplier Multiplier steering the intervention strength
- noiseMult Multiplier steering the noise level
- fracVarInt If knownInterventions was set to TRUE, fraction of variables that were intervened on in each environment.
- hiddenVars If TRUE, hidden variables exist.
- knownInterventions If TRUE, location of interventions is known.
- simulateObs If TRUE, environment 1 contains observational data.

**References**

Dominik Rothenhaeusler, Christina Heinze, Jonas Peters, Nicolai Meinshausen (2015): backShift: Learning causal cyclic graphs from unknown shift interventions. arXiv preprint: <http://arxiv.org/abs/1506.02494>

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