

# Package ‘bspcov’

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**Title** Bayesian Sparse Estimation of a Covariance Matrix

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**Description** Provides functions which perform Bayesian estimations of a covariance matrix for multivariate normal data. Assumes that the covariance matrix is sparse or band matrix and positive-definite. This software has been developed using funding supported by Basic Science Research Program through the National Research Foundation of Korea ('NRF') funded by the Ministry of Education ('RS-2023-00211979', 'NRF-2022R1A5A7033499', 'NRF-2020R1A4A1018207' and 'NRF-2020R1C1C1A01013338').

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bandPPP

*Bayesian Estimation of a Banded Covariance Matrix*

---

### Description

Provides a post-processed posterior for Bayesian inference of a banded covariance matrix.

### Usage

```
bandPPP(X, k, eps, prior = list(), nsample = 2000)
```

### Arguments

- X            a  $n \times p$  data matrix with column mean zero.
- k            a scalar value (natural number) specifying the bandwidth of covariance matrix.
- eps          a small positive number decreasing to 0 with default value  $(\log(k))^2 * (k + \log(p))/n$ .
- prior        a list giving the prior information. The list includes the following parameters (with default values in parentheses): A (I) giving the positive definite scale matrix for the inverse-Wishart prior, nu ( $p + k$ ) giving the degree of freedom of the inverse-Wishart prior.
- nsample      a scalar value giving the number of the post-processed posterior samples.

## Details

Lee, Lee, and Lee (2023+) proposed a two-step procedure generating samples from the post-processed posterior for Bayesian inference of a banded covariance matrix:

- Initial posterior computing step: Generate random samples from the following initial posterior obtained by using the inverse-Wishart prior  $IW_p(B_0, \nu_0)$

$$\Sigma | X_1, \dots, X_n \sim IW_p(B_0 + nS_n, \nu_0 + n),$$

$$\text{where } S_n = n^{-1} \sum_{i=1}^n X_i X_i^\top.$$

- Post-processing step: Post-process the samples generated from the initial samples

$$\Sigma^{(i)} := \begin{cases} B_k(\Sigma^{(i)}) + [\epsilon_n - \lambda_{\min}\{B_k(\Sigma^{(i)})\}] I_p, & \text{if } \lambda_{\min}\{B_k(\Sigma^{(i)})\} < \epsilon_n, \\ B_k(\Sigma^{(i)}), & \text{otherwise,} \end{cases}$$

where  $\Sigma^{(1)}, \dots, \Sigma^{(N)}$  are the initial posterior samples,  $\epsilon_n$  is a small positive number decreasing to 0 as  $n \rightarrow \infty$ , and  $B_k(B)$  denotes the  $k$ -band operation given as

$$B_k(B) = \{b_{ij} I(|i - j| \leq k)\} \text{ for any } B = (b_{ij}) \in R^{p \times p}.$$

For more details, see Lee, Lee and Lee (2023+).

## Value

Sigma	a nsample $\times$ p(p+1)/2 matrix including lower triangular elements of covariance matrix.
p	dimension of covariance matrix.

## Author(s)

Kwangmin Lee

## References

Lee, K., Lee, K., and Lee, J. (2023+), "Post-processes posteriors for banded covariances", *Bayesian Analysis*, DOI: 10.1214/22-BA1333.

## See Also

cv.bandPPP estimate

## Examples

```
n <- 25
p <- 50
Sigma0 <- diag(1, p)
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = Sigma0)
res <- bspcov::bandPPP(X, 2, 0.01, nsample=100)
```

---

bmmpcov*Bayesian Sparse Covariance Estimation*

---

## Description

Provides a Bayesian sparse and positive definite estimate of a covariance matrix via the beta-mixture shrinkage prior.

## Usage

```
bmmpcov(X, Sigma, prior = list(), nsample = list())
```

## Arguments

X	a $n \times p$ data matrix with column mean zero.
Sigma	an initial guess for Sigma.
prior	a list giving the prior information. The list includes the following parameters (with default values in parentheses): a (1/2) and b (1/2) giving the shape parameters for beta distribution, lambda (1) giving the hyperparameter for the diagonal elements, tau1sq (10000/(n*p^4)) giving the hyperparameter for the shrinkage prior of covariance.
nsample	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): burnin (1000) giving the number of MCMC samples in transition period, nmc (1000) giving the number of MCMC samples for analysis.

## Details

Lee, Jo and Lee (2022) proposed the beta-mixture shrinkage prior for estimating a sparse and positive definite covariance matrix. The beta-mixture shrinkage prior for  $\Sigma = (\sigma_{jk})$  is defined as

$$\pi(\Sigma) = \frac{\pi^u(\Sigma)I(\Sigma \in C_p)}{\pi^u(\Sigma \in C_p)}, \quad C_p = \{ \text{all } p \times p \text{ positive definite matrices} \},$$

where  $\pi^u(\cdot)$  is the unconstrained prior given by

$$\pi^u(\sigma_{jk} | \rho_{jk}) = N\left(\sigma_{jk} | 0, \frac{\rho_{jk}}{1 - \rho_{jk}} \tau_1^2\right)$$

$$\pi^u(\rho_{jk}) = Beta(\rho_{jk} | a, b), \quad \rho_{jk} = 1 - 1/(1 + \phi_{jk})$$

$$\pi^u(\sigma_{jj}) = Exp(\sigma_{jj} | \lambda).$$

For more details, see Lee, Jo and Lee (2022).

**Value**

Sigma	a $nmc \times p(p+1)/2$ matrix including lower triangular elements of covariance matrix.
Phi	a $nmc \times p(p+1)/2$ matrix including shrinkage parameters corresponding to lower triangular elements of covariance matrix.
p	dimension of covariance matrix.

**Author(s)**

Kyoungjae Lee and Seongil Jo

**References**

Lee, K., Jo, S., and Lee, J. (2022), "The beta-mixture shrinkage prior for sparse covariances with near-minimax posterior convergence rate", *Journal of Multivariate Analysis*, 192, 105067.

**See Also**

sbmspcov

**Examples**

```

set.seed(1)
n <- 20
p <- 5

# generate a sparse covariance matrix:
True.Sigma <- matrix(0, nrow = p, ncol = p)
diag(True.Sigma) <- 1
Values <- -runif(n = p*(p-1)/2, min = 0.2, max = 0.8)
nonzeroIND <- which(rbinom(n=p*(p-1)/2, 1, prob=1/p)==1)
zeroIND = (1:(p*(p-1)/2))[-nonzeroIND]
Values[zeroIND] <- 0
True.Sigma[lower.tri(True.Sigma)] <- Values
True.Sigma[upper.tri(True.Sigma)] <- t(True.Sigma)[upper.tri(True.Sigma)]
if(min(eigen(True.Sigma)$values) <= 0){
  delta <- -min(eigen(True.Sigma)$values) + 1.0e-5
  True.Sigma <- True.Sigma + delta*diag(p)
}

# generate a data
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = True.Sigma)

# compute sparse, positive covariance estimator:
fout <- bspcov::bmspcov(X = X, Sigma = diag(diag(cov(X))))
post.est.m <- bspcov::estimate(fout)
sqrt(mean((post.est.m - True.Sigma)^2))
sqrt(mean((cov(X) - True.Sigma)^2))

```

colon	<i>colon dataset</i>
-------	----------------------

### Description

The colon cancer dataset, which includes gene expression values from 22 colon tumor tissues and 40 non-tumor tissues.

### Format

'data.frame'

### Source

<http://genomics-pubs.princeton.edu/oncology/affydata/>.

### Examples

```
data("colon")
head(colon)
```

cv.bandPPP	<i>CV for Bayesian Estimation of a Banded Covariance Matrix</i>
------------	---

### Description

Performs leave-one-out cross-validation (LOOCV) to calculate the predictive log-likelihood for a post-processed posterior of a banded covariance matrix and selects the optimal parameters.

### Usage

```
cv.bandPPP(X, kvec, epsvec, prior = list(), nsample = 2000, ncores = 2)
```

### Arguments

X	a $n \times p$ data matrix with column mean zero.
kvec	a vector of natural numbers specifying the bandwidth of covariance matrix.
epsvec	a vector of small positive numbers decreasing to 0.
prior	a list giving the prior information. The list includes the following parameters (with default values in parentheses): A (I) giving the positive definite scale matrix for the inverse-Wishart prior, nu ( $p + k$ ) giving the degree of freedom of the inverse-Wishart prior.
nsample	a scalar value giving the number of the post-processed posterior samples.
ncores	a scalar value giving the number of CPU cores.

## Details

The predictive log-likelihood for each  $k$  and  $\epsilon_n$  is estimated as follows:

$$\sum_{i=1}^n \log S^{-1} \sum_{s=1}^S p(X_i | B_k^{(\epsilon_n)}(\Sigma_{i,s})),$$

where  $X_i$  is the  $i$ th observation,  $\Sigma_{i,s}$  is the  $s$ th posterior sample based on  $(X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_n)$ , and  $B_k^{(\epsilon_n)}$  represents the banding post-processing function. For more details, see (3) in Lee, Lee and Lee (2023+).

## Value

elpd	a $M \times 3$ dataframe having the expected log predictive density (ELPD) for each $k$ and $\text{eps}$ , where $M = \text{length}(\text{kvec}) * \text{length}(\text{epsvec})$ .
------	--

## Author(s)

Kwangmin Lee

## References

Lee, K., Lee, K., and Lee, J. (2023+), "Post-processes posteriors for banded covariances", *Bayesian Analysis*, DOI: 10.1214/22-BA1333.

Gelman, A., Hwang, J., and Vehtari, A. (2014). "Understanding predictive information criteria for Bayesian models." *Statistics and computing*, 24(6), 997-1016.

## See Also

bandPPP

## Examples

```

Sigma0 <- diag(1,50)
X <- mvtnorm::rmvnorm(25,sigma = Sigma0)
kvec <- 1:2
epsvec <- c(0.01,0.05)
res <- bspcov::cv.bandPPP(X,kvec,epsvec,nsample=10,ncores=4)
plot(res)

```

## Description

Performs cross-validation to estimate spectral norm error for a post-processed posterior of a sparse covariance matrix.

## Usage

```
cv.thresPPP(
  X,
  thresvec,
  epsvec,
  prior = NULL,
  thresfun = "hard",
  nsample = 2000,
  ncores = 2
)
```

## Arguments

X	a $n \times p$ data matrix with column mean zero.
thresvec	a vector of real numbers specifying the parameter of the threshold function.
epsvec	a vector of small positive numbers decreasing to 0.
prior	a list giving the prior information. The list includes the following parameters (with default values in parentheses): A (I) giving the positive definite scale matrix for the inverse-Wishart prior, nu ( $p + k$ ) giving the degree of freedom of the inverse-Wishart prior.
thresfun	a string to specify the type of threshold function. fun ('hard') giving the thresholding function ('hard' or 'soft') for the thresholding PPP procedure.
nsample	a scalar value giving the number of the post-processed posterior samples.
ncores	a scalar value giving the number of CPU cores.

## Details

Given a set of train data and validation data, the spectral norm error for each  $\gamma$  and  $\epsilon_n$  is estimated as follows:

$$\|\hat{\Sigma}(\gamma, \epsilon_n)^{(train)} - S^{(val)}\|_2$$

where  $\hat{\Sigma}(\gamma, \epsilon_n)^{(train)}$  is the estimate for the covariance based on the train data and  $S^{(val)}$  is the sample covariance matrix based on the validation data. The spectral norm error is estimated by the 10-fold cross-validation. For more details, see the first paragraph on page 9 in Lee and Lee (2023).

**Value**

CVdf a  $M \times 3$  dataframe having the estimated spectral norm error for each thres and eps, where  $M = \text{length}(\text{thresvec}) * \text{length}(\text{epsvec})$

**Author(s)**

Kwangmin Lee

**References**

Lee, K. and Lee, J. (2023), "Post-processes posteriors for sparse covariances", *Journal of Econometrics*, 236(3), 105475.

**See Also**

thresPPP

**Examples**

```
Sigma0 <- diag(1,50)
X <- mvtnorm::rmvnorm(25,sigma = Sigma0)
thresvec <- c(0.01,0.1)
epsvec <- c(0.01,0.1)
res <- bspcov::cv.thresPPP(X,thresvec,epsvec,nsample=100)
plot(res)
```

---

estimate

*Point-estimate of posterior distribution*

---

**Description**

Compute the point estimate (mean) to describe posterior distribution.

**Usage**

```
estimate(object, ...)
## S3 method for class 'bspcov'
estimate(object, ...)
```

**Arguments**

object an object from **bandPPP**, **bmspcov**, **smbmspcov**, and **thresPPP**.  
... additional arguments for estimate.

**Value**

**Sigma** the point estimate (mean) of covariance matrix.

**Author(s)**

Seongil Jo

**See Also**

`plot.postmean.bspcov`

**Examples**

```
n <- 25
p <- 50
Sigma0 <- diag(1, p)
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = Sigma0)
res <- bspcov::bandPPP(X, 2, 0.01, nsample=100)
est <- bspcov::estimate(res)
```

**plot.bspcov**

*Plot Diagnostics of Posterior Samples and Cross-Validation*

**Description**

Provides a trace plot of posterior samples and a plot of a learning curve for cross-validation

**Usage**

```
## S3 method for class 'bspcov'
plot(x, ..., cols, rows)
```

**Arguments**

<b>x</b>	an object from <b>bmspcov</b> , <b>sbmspcov</b> , <b>cv.bandPPP</b> , and <b>cv.thresPPP</b> .
<b>...</b>	additional arguments for ggplot2.
<b>cols</b>	a scalar or a vector including specific column indices for the trace plot.
<b>rows</b>	a scalar or a vector including specific row indices greater than or equal to columns indices for the trace plot.

**Value**

**plot** a plot for diagnostics of posterior samples **x**.

**Author(s)**

Seongil Jo

## Examples

```

set.seed(1)
n <- 100
p <- 20

# generate a sparse covariance matrix:
True.Sigma <- matrix(0, nrow = p, ncol = p)
diag(True.Sigma) <- 1
Values <- -runif(n = p*(p-1)/2, min = 0.2, max = 0.8)
nonzeroIND <- which(rbinom(n=p*(p-1)/2, 1, prob=1/p)==1)
zeroIND = (1:(p*(p-1)/2))[-nonzeroIND]
Values[zeroIND] <- 0
True.Sigma[lower.tri(True.Sigma)] <- Values
True.Sigma[upper.tri(True.Sigma)] <- t(True.Sigma)[upper.tri(True.Sigma)]
if(min(eigen(True.Sigma)$values) <= 0){
  delta <- -min(eigen(True.Sigma)$values) + 1.0e-5
  True.Sigma <- True.Sigma + delta*diag(p)
}

# generate a data
X <- MASS:::mvrnorm(n = n, mu = rep(0, p), Sigma = True.Sigma)

# compute sparse, positive covariance estimator:
fout <- bspcov::sbmspcov(X = X, Sigma = diag(diag(cov(X))))
plot(fout, cols = c(1, 3, 4), rows = c(1, 3, 4))
plot(fout, cols = 1, rows = 1:3)

# Cross-Validation for Banded Structure
Sigma0 <- diag(1,50)
X <- mvtnorm::rmvnorm(25,sigma = Sigma0)
kvec <- 1:2
epsvec <- c(0.01,0.05)
res <- bspcov::cv.bandPPP(X,kvec,epsvec,nsample=10,ncores=4)
plot(res)

```

`plot.postmean.bspcov` *Draw a Heat Map for Point Estimate of Covariance Matrix*

## Description

Provides a heat map for posterior mean estimate of sparse covariance matrix

## Usage

```
## S3 method for class 'postmean.bspcov'
plot(x, ...)
```

**Arguments**

- `x` an object from **estimate**.
- `...` additional arguments for ggplot2.

**Value**

- `plot` a heatmap for point estimate of covariance matrix `x`.

**Author(s)**

Seongil Jo

**See Also**

`estimate`

**Examples**

```
n <- 25
p <- 50
Sigma0 <- diag(1, p)
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = Sigma0)
res <- bspcov::thresPPP(X, eps=0.01, thres=list(value=0.5,fun='hard'), nsample=100)
est <- bspcov::estimate(res)
plot(est)
```

**Description**

Provides a Bayesian sparse and positive definite estimate of a covariance matrix via screened beta-mixture prior.

**Usage**

```
sbmspcov(X, Sigma, cutoff = list(), prior = list(), nsample = list())
```

**Arguments**

- `X` a  $n \times p$  data matrix with column mean zero.
- `Sigma` an initial guess for `Sigma`.

cutoff	a list giving the information for the threshold. The list includes the following parameters (with default values in parentheses): method ('FNR') giving the method for determining the threshold value (method='FNR' uses the false negative rate (FNR)-based approach, method='corr' chooses the threshold value by sample correlations), rho a lower bound of meaningfully large correlations whose the defaults values are 0.25 and 0.2 for method = 'FNR' and method = 'corr', respectively. Note. If method='corr', rho is used as the threshold value.
prior	a list giving the prior information. The list includes the following parameters (with default values in parentheses): a (1/2) and b (1/2) giving the shape parameters for beta distribution, lambda (1) giving the hyperparameter for the diagonal elements, tau1sq ( $\log(p)/(p^2 \cdot n)$ ) giving the hyperparameter for the shrinkage prior of covariance.
nsample	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): burnin (1000) giving the number of MCMC samples in transition period, nmc (1000) giving the number of MCMC samples for analysis.

## Details

Lee, Jo, Lee, and Lee (2023+) proposed the screened beta-mixture shrinkage prior for estimating a sparse and positive definite covariance matrix. The screened beta-mixture shrinkage prior for  $\Sigma = (\sigma_{jk})$  is defined as

$$\pi(\Sigma) = \frac{\pi^u(\Sigma)I(\Sigma \in C_p)}{\pi^u(\Sigma \in C_p)}, \quad C_p = \{ \text{all } p \times p \text{ positive definite matrices} \},$$

where  $\pi^u(\cdot)$  is the unconstrained prior given by

$$\begin{aligned} \pi^u(\sigma_{jk} | \psi_{jk}) &= N\left(\sigma_{jk} | 0, \frac{\psi_{jk}}{1 - \psi_{jk}}\tau_1^2\right), \quad \psi_{jk} = 1 - 1/(1 + \phi_{jk}) \\ \pi^u(\psi_{jk}) &= Beta(\psi_{jk} | a, b) \text{ for } (j, k) \in S_r(\hat{R}) \\ \pi^u(\sigma_{jj}) &= Exp(\sigma_{jj} | \lambda), \end{aligned}$$

where  $S_r(\hat{R}) = \{(j, k) : 1 < j < k \leq p, |\hat{\rho}_{jk}| > r\}$ ,  $\hat{\rho}_{jk}$  are sample correlations, and  $r$  is a threshold given by user.

For more details, see Lee, Jo, Lee and Lee (2022+).

## Value

Sigma	a nmc $\times$ p(p+1)/2 matrix including lower triangular elements of covariance matrix.
p	dimension of covariance matrix.
Phi	a nmc $\times$ p(p+1)/2 matrix including shrinkage parameters corresponding to lower triangular elements of covariance matrix.
INDzero	a list including indices of off-diagonal elements screened by sure screening.
cutoff	the cutoff value specified by FNR-approach.

**Author(s)**

Kyoungjae Lee and Seongil Jo

**References**

Lee, K., Jo, S., Lee, K., and Lee, J. (2023+), "Scalable and optimal Bayesian inference for sparse covariance matrices via screened beta-mixture prior", arXiv:2206.12773.

**See Also**

bmspcov

**Examples**

```

set.seed(1)
n <- 20
p <- 5

# generate a sparse covariance matrix:
True.Sigma <- matrix(0, nrow = p, ncol = p)
diag(True.Sigma) <- 1
Values <- -runif(n = p*(p-1)/2, min = 0.2, max = 0.8)
nonzeroIND <- which(rbinom(n=p*(p-1)/2,1,prob=1/p)==1)
zeroIND = (1:(p*(p-1)/2))[-nonzeroIND]
Values[zeroIND] <- 0
True.Sigma[lower.tri(True.Sigma)] <- Values
True.Sigma[upper.tri(True.Sigma)] <- t(True.Sigma)[upper.tri(True.Sigma)]
if(min(eigen(True.Sigma)$values) <= 0){
  delta <- -min(eigen(True.Sigma)$values) + 1.0e-5
  True.Sigma <- True.Sigma + delta*diag(p)
}

# generate a data
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = True.Sigma)

# compute sparse, positive covariance estimator:
fout <- bspcov::sbmspcov(X = X, Sigma = diag(diag(cov(X))))
post.est.m <- bspcov::estimate(fout)
sqrt(mean((post.est.m - True.Sigma)^2))
sqrt(mean((cov(X) - True.Sigma)^2))

```

**Description**

The idiosyncratic error of S&P 500 data

**Format**

'list'

**Source**

State Street Global Advisors (2022).

**Examples**

```
data("SP500_idioerr")
names(SP500_idioerr)
```

---

summary.bspcov*Summary of Posterior Distribution*

---

**Description**

Provides the summary statistics for posterior samples of covariance matrix.

**Usage**

```
## S3 method for class 'bspcov'
summary(object, cols, rows, ...)
```

**Arguments**

object	an object from <b>bandPPP</b> , <b>bmspcov</b> , <b>sbmmspcov</b> , and <b>thresPPP</b> .
cols	a scalar or a vector including specific column indices.
rows	a scalar or a vector including specific row indices greater than or equal to columns indices.
...	additional arguments for the summary function.

**Value**

summary	a table of summary statistics including empirical mean, standard deviation, and quantiles for posterior samples
---------	---

**Note**

If both `cols` and `rows` are vectors, they must have the same length.

**Author(s)**

Seongil Jo

## Examples

```

set.seed(1)
n <- 20
p <- 5

# generate a sparse covariance matrix:
True.Sigma <- matrix(0, nrow = p, ncol = p)
diag(True.Sigma) <- 1
Values <- -runif(n = p*(p-1)/2, min = 0.2, max = 0.8)
nonzeroIND <- which(rbinom(n=p*(p-1)/2, 1, prob=1/p)==1)
zeroIND = (1:(p*(p-1)/2))[-nonzeroIND]
Values[zeroIND] <- 0
True.Sigma[lower.tri(True.Sigma)] <- Values
True.Sigma[upper.tri(True.Sigma)] <- t(True.Sigma)[upper.tri(True.Sigma)]
if(min(eigen(True.Sigma)$values) <= 0){
  delta <- -min(eigen(True.Sigma)$values) + 1.0e-5
  True.Sigma <- True.Sigma + delta*diag(p)
}

# generate a data
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = True.Sigma)

# compute sparse, positive covariance estimator:
fout <- bspcov::sbmspcov(X = X, Sigma = diag(diag(cov(X))))
summary(fout, cols = c(1, 3, 4), rows = c(1, 3, 4))
summary(fout, cols = 1, rows = 1:p)

```

## Description

Provides a post-processed posterior (PPP) for Bayesian inference of a sparse covariance matrix.

## Usage

```
thresPPP(X, eps, thres = list(), prior = list(), nsample = 2000)
```

## Arguments

- |       |   |
|-------|---|
| X     | a $n \times p$ data matrix with column mean zero.   |
| eps   | a small positive number decreasing to 0.  |
| thres | a list giving the information for thresholding PPP procedure. The list includes the following parameters (with default values in parentheses): value (0.1) giving the positive real number for the thresholding PPP procedure, fun ('hard') giving the thresholding function ('hard' or 'soft') for the thresholding PPP procedure. |

prior	a list giving the prior information. The list includes the following parameters (with default values in parentheses): A (I) giving the positive definite scale matrix for the inverse-Wishart prior, nu (p + 1) giving the degree of freedom of the inverse-Wishart prior.
nsample	a scalar value giving the number of the post-processed posterior samples.

## Details

Lee and Lee (2023) proposed a two-step procedure generating samples from the post-processed posterior for Bayesian inference of a sparse covariance matrix:

- Initial posterior computing step: Generate random samples from the following initial posterior obtained by using the inverse-Wishart prior  $IW_p(B_0, \nu_0)$

$$\Sigma | X_1, \dots, X_n \sim IW_p(B_0 + nS_n, \nu_0 + n),$$

where  $S_n = n^{-1} \sum_{i=1}^n X_i X_i^\top$ .

- Post-processing step: Post-process the samples generated from the initial samples

$$\Sigma^{(i)} := \begin{cases} H_{\gamma_n}(\Sigma^{(i)}) + [\epsilon_n - \lambda_{\min}\{H_{\gamma_n}(\Sigma^{(i)})\}] I_p, & \text{if } \lambda_{\min}\{H_{\gamma_n}(\Sigma^{(i)})\} < \epsilon_n, \\ H_{\gamma_n}(\Sigma^{(i)}), & \text{otherwise,} \end{cases}$$

where  $\Sigma^{(1)}, \dots, \Sigma^{(N)}$  are the initial posterior samples,  $\epsilon_n$  is a positive real number, and  $H_{\gamma_n}(\Sigma)$  denotes the generalized thresholding operator given as

$$(H_{\gamma_n}(\Sigma))_{ij} = \begin{cases} \sigma_{ij}, & \text{if } i = j, \\ h_{\gamma_n}(\sigma_{ij}), & \text{if } i \neq j, \end{cases}$$

where  $\sigma_{ij}$  is the  $(i, j)$  element of  $\Sigma$  and  $h_{\gamma_n}(\cdot)$  is a generalized thresholding function.

For more details, see Lee and Lee (2023).

## Value

Sigma	a nsample $\times$ p(p+1)/2 matrix including lower triangular elements of covariance matrix.
p	dimension of covariance matrix.

## Author(s)

Kwangmin Lee

## References

Lee, K. and Lee, J. (2023), "Post-processed posteriors for sparse covariances", *Journal of Econometrics*.

## See Also

cv.thresPPP

**Examples**

```
n <- 25
p <- 50
Sigma0 <- diag(1, p)
X <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = Sigma0)
res <- bspcov::thresPPP(X, eps=0.01, thres=list(value=0.5,fun='hard')), nsample=100)
est <- bspcov::estimate(res)
```

*tissues**tissues dataset***Description**

The *tissues* data of colon cancer dataset, which includes gene expression values from 22 colon tumor tissues and 40 non-tumor tissues.

**Format**

‘numeric’

**Source**

<http://genomics-pubs.princeton.edu/oncology/affydata/>.

**Examples**

```
data("tissues")
head(tissues)
```

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